

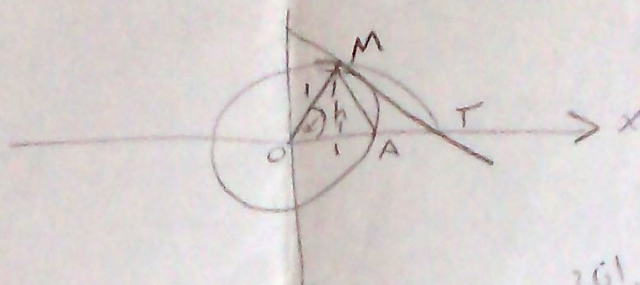
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(1)

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

نظرية

البرهان

فرضنا ان $0 < x < \pi/2$



من الشكل نجد ان:

$$\Delta P_{OMA} < \Delta P_{OMA} < \Delta P_{OMT}$$

$$\frac{1}{2}(1)h < \frac{1}{2}(1)x < \frac{1}{2}(1)MT$$

$$h < x < MT$$

$$\therefore \tan = \frac{MT}{1}$$

$$\therefore \sin = \frac{h}{1}$$

$$\therefore \sin x < x < \tan x \rightarrow [1]$$

بالقسمة على $\sin x$

$$1 < \frac{x}{\sin x} < \cos x \rightarrow [2]$$

$$\therefore \cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$\therefore \sin x < x \rightarrow \sin \frac{x}{2} < \frac{x}{2}$$

$$\rightarrow \sin^2 \frac{x}{2} < \frac{x^2}{4} \rightarrow 2\sin^2 \frac{x}{2} < \frac{x^2}{2} \rightarrow 1 - 2\sin^2 \frac{x}{2} < 1 - \frac{x^2}{2}$$

$$\rightarrow \cos x < 1 - \frac{x^2}{2}$$

$$\therefore 1 < \frac{x}{\sin x} < \cos x < 1 - \frac{x^2}{2}$$

$$\lim_{x \rightarrow 0} \frac{1-x^2}{2} = 1$$

$$1 < \sin x < 1$$

2

$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$$

$$\lim_{x \rightarrow 0} x = 0 \quad \sin \frac{1}{x} \text{ oscillates}$$

$$\therefore \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = 0$$

$$\lim_{x \rightarrow 0} \frac{\sin \alpha x^3}{\sin \beta x^3}$$

$$\frac{\sin \alpha x^3 / x^3}{\sin \beta x^3 / x^3} = \frac{\alpha}{\beta}$$

$$\lim_{x \rightarrow 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{\sin x}$$

$$\lim_{x \rightarrow 0} x^2 = 0 \quad \therefore \sin\left(\frac{1}{x}\right) \text{ oscillates}$$

$$\frac{0}{1} = 0$$

$$\lim_{x \rightarrow 0} \frac{\tan^2 Kx + \sin^2 mx}{x^2}$$

$$= K^2 + m^2$$

$$\lim_{x \rightarrow a} f(x) = b_1, \quad \lim_{x \rightarrow a} f(x) = b_2, \quad b_1, b_2 \in \mathbb{R}$$

$$\forall \varepsilon > 0 \exists \delta(\varepsilon) > 0 \ni \forall x \in G$$

$$0 < |x - a| < \delta \rightarrow |f(x) - b_1| < \varepsilon \rightarrow \text{I}$$

$$\forall \varepsilon > 0 \exists \delta(\varepsilon) > 0 \ni \forall x \in G$$

$$0 < |x - a| < \delta \rightarrow |f(x) - b_2| < \varepsilon \rightarrow \text{II}$$

$$\delta = \min(\delta_1, \delta_2) \text{ باختيار}$$

$$\therefore \forall x \in G, 0 < |x - a| < \delta$$

$$\leq |b_1 - b_2| \leq |b_1 - f(x) + f(x) - b_2| \leq |b_1 - f(x)| + |f(x) - b_2|$$

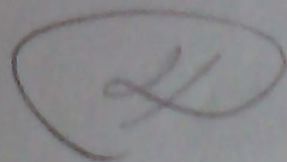
$$\varepsilon + \varepsilon = 2\varepsilon$$

$$0 \leq |b_1 - b_2| < 2\varepsilon$$

$\therefore \varepsilon > 0$ عدد صغير جدا جدا

$$\therefore b_1 - b_2 = 0$$

$$\therefore b_1 = b_2$$



4) f, g, h ثلاث دوال معرفة على G

$$\text{iii) } f(x) \leq g(x) \leq h(x)$$

$$\text{ii) } \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = b$$

$$\therefore \lim_{x \rightarrow a} g(x) = b$$

افرضنا ان

ونحقق ان

$$b \in \mathbb{R}$$

البرهان

$$\therefore \lim_{x \rightarrow a} f(x) = b$$

$$\therefore \forall \varepsilon > 0 \exists \delta(\varepsilon) > 0 \rightarrow \forall x \in G$$

$$0 < |x - a| < \delta \rightarrow |f(x) - b| < \varepsilon \rightarrow \text{ii}$$

$$\therefore \lim_{x \rightarrow a} h(x) = b$$

$$\therefore \forall \varepsilon > 0 \exists \delta(\varepsilon) > 0 \rightarrow \forall x \in G$$

$$0 < |x - a| < \delta \rightarrow |h(x) - b| < \varepsilon \rightarrow \text{ii}$$

بالتالي $\delta = \min(\delta_1, \delta_2)$

$$\forall x \in G, 0 < |x - a| < \delta \Rightarrow |f(x) - b| < \varepsilon \text{ and } |h(x) - b| < \varepsilon$$

$$\Rightarrow -\varepsilon < f(x) - b < \varepsilon \text{ and } -\varepsilon < h(x) - b < \varepsilon$$

$$\therefore f(x) \leq g(x) \leq h(x)$$

$$-\varepsilon < f(x) - b \leq g(x) - b \leq h(x) - b < \varepsilon$$

$$\forall x \in G, 0 < |x - a| < \delta \rightarrow -\varepsilon < g(x) - b < \varepsilon$$

$$\Rightarrow |g(x) - b| < \varepsilon$$

$$\therefore \lim_{x \rightarrow a} g(x) = b$$

